Abstract

This paper presents an overview of a research of the control of a gantry crane for containers. This program includes the modeling of the dynamic behavior of a container crane and the computation of the time-optimal trajectories for transporting the container. A control system has been designed according the Model-Based Predictive Control principle (MBPC). This controller is applied for generating and following time-optimal trajectories for the transport of the container without swing and skew. The controller takes care for a fast and stable behavior of the container in its end position within the required accuracy. The controller takes also into account the constraints on maximum acceleration and speed and obstacles between quay and container vessel.

Keywords: container crane, modeling, non-linear control, optimization, trajectory

1 INTRODUCTION

During 1996, the Centre of Transport Technology (CTT) has initiated the FAMAS (First, All Modes, All Sizes) programme. The goal of the programme is to develop a new generation of container terminals capable to handle all modalities of transhipment of all container sizes with an equal service level. This concerns all modalities, including the coming Jumbo Container Vessel with a capacity of eight to ten thousand TEU (Twenty-foot Equivalent Unit). To execute this programme a consortium has been formed by several major, transport related companies like ECT, Siemens Nederland, Nelcon and the Delft University of Technology. To accomplish the goal of the programme, a highly automated terminal has to be developed. Although, robotising is not the main goal of the programme, it is the opinion that handling containers with a throughput of 500,000 TEU a year or more can only be done economically and efficiently by robotising stacking and terminal transport. The programme is subdivided into several projects, where New Terminal Control (NewCon), Jumbo Container Crane (JCC) and Automated Guided Vehicles (AGV) are the most import projects. New technologies will be developed or enhanced like anti-sway and anti-skew controllers, stacking algorithms, AGV control and terminal information techniques.

The demands made on container handling in international seaports are continuously increasing, because both the capacity of the vessels is increasing, and the service time of container vessels should be decreasing. In order to minimise this service time a new generation of the so-called Jumbo Container Cranes will be developed. This paper presents a control system for this new generation of container gantry cranes. The requirements for this control system are to:

- accurately position the container in its end position from an arbitrary starting position and initial conditions.
- transport the container in a time optimal way.
- avoid large overshoot in container position.
- operate robust e.g. independent of disturbances especially of wind forces.
- take into account constraints on maximum force, acceleration and speed.
The control system applies a state feedback loop with time variable gains. The feedback gains are continuously adjusted using a model based optimisation strategy in order to optimize the performance of the container crane for all possible initial conditions, container masses, and cable lengths. The control system activates the drive systems of the crane such that a container is transported in a time-optimal way, and is positioned accurately and robustly on the desired destination. Section 2 presents the container gantry crane models. In section 3 the methods of determining the container trajectories are given. Section 4 presents the design of the controller and in section 5 simulation results are shown.

2 MODELLING

During the last quarter of the former century various models for cranes are developed. These models can be distinguished by different complexity of modelling and by the nature of the neglected parameters. One can distinguish:
- models with constant and variable cable length
- models including non-linearities or containing linearized equations,
- models taking into account disturbances like wind.
Simple models enable easier and more mathematical analysis and give better insight in the design and the possibilities of different control algorithms and leads to better insight in robustness and stability. At the other hand more complex models are necessary to approximate the reality closer. However, it is impossible to include all effects of real life in a mathematical model. Various classes of friction, (viscous friction, coulomb friction but in particular stiction) are difficult to model but also the elasticity of the cables and the flexibility of the crane construction will have its influence on the behaviour of the controller. There will always remain differences between the simulated control system and that of real crane.
Depending on the control variable, there is a need for investigating two-dimensional (2D) or three-dimension (3D) models. The majority of the models are related to the control of sway of the container where a two-dimensional model usually will be sufficient. But for more complex dynamical movements of the container like the combination of swing and skew 3D or quasi-3D models become necessary for designing the controller.
In practice it is impossible to do experimental research on a real-size crane. To facilitate the transfer of a controller a computer environment to a controller of a real crane, it is desirable to develop a mechanical laboratory model of the crane. With this laboratory model, which is a scaled model of the real crane, a variety of control algorithms can be studied in a more realistic environment.

*Container Gantry Crane Model*

Figure 1 presents a schematic representation of the container gantry crane. In the model the mass of the load (container and spreader) is concentrated in a single point, that is suspended to the trolley by a weightless cable. Dissipation (like friction), and elastic deformation of the crane construction and the cables have been neglected.
The following variables are used for all models:

- $F_c$ force acting on container in horizontal direction
- $F_h$ hoisting drive system force
- $F_t$ trolley drive system force
- $g$ gravitational constant (9.81 m/s$^2$)
- $J_h,J_t$ moment of inertia of all rotating parts of hoisting and trolley drive system, respectively
- $h$ cable length
- $m_c$ mass of load (spreader+container)
- $m_t$ mass of trolley
- $m_{t,eff}$ effective mass of trolley $m_{t,eff} = m_t + J_t/(N_t/r_t)^2$
- $N_h,N_t$ gear box ratio of hoisting and trolley drive system, respectively
- $r_h,r_t$ radius of cable drum of the hoisting and trolley drive system, respectively
- $x_c$ horizontal container position
- $x_t$ trolley position
- $\theta$ swing angle

**Non-linear Model**

This section presents two varieties of two-dimensional dynamical models of a container gantry crane. The non-linear model is used for simulating the container crane and the linear model is used for analysis and synthesis of the control system.

The generalised coordinates of this model are $x_t$, $x_c$, and $\ell$. Formulating the Lagrange equation for the system using these coordinates, and solving for $\ddot{x}_t$, $\ddot{x}_c$, and $\ddot{\ell}$ yields a sixth-order state space model:

\[
\ddot{x}_t = \frac{F_c \ell - F_h \Delta}{m_{t,eff} \ell} \tag{1}
\]

\[
\ddot{x}_c = \frac{F_c \ell + F_h \Delta}{m_t \ell} \tag{2}
\]
\[ \ell = \sum_{i=1}^{4} L_i \]  
\[ \text{(3)} \]

with:

\[ L_i = F_i \left( m_{t,\text{eff}} - \Delta^4 \ell^4 \right) / \ell^2 m_{t,\text{eff}} \xi^2 \]

\[ L_2 = \Delta^2 \ell \left( -\rho \Delta + m_{t,\text{eff}} \ell \right) / \ell^2 m_{t,\text{eff}} \xi^2 \]

\[ L_3 = \Delta^2 \left( -F_i \lambda \left( m_{t,\text{eff}} + m_i \right) - 2m_{t,\text{eff}} \ell \right) / \ell m_{t,\text{eff}} m_i \xi^2 \]

\[ L_4 = \left( \rho_{\text{eff}} \Delta + m_{t,\text{eff}} \ell \right) / \ell m_{t,\text{eff}} m_i \xi^2 \]

\[ \Delta = x_i - x_i, \quad \xi = \sqrt{\ell^2 - \Delta^2} \]

\[ \rho = F_i m_i - F_i m_i, \quad \rho_{\text{eff}} = F_i m_{t,\text{eff}} - F_i m_i \]

**Linear model**

In order to obtain a linear model the cable length is assumed to be constant. The generalised coordinates of this model are \( x_i \) and \( \theta \). Formulating the Lagrange equation for the system using these coordinates, solving for \( \dot{x}_i \) and \( \dot{\theta} \), and linearizing by approximating \( \sin \theta = \theta \) and \( \cos \theta = 1 \) yields a fourth-order state space model:

\[
\begin{bmatrix}
\dot{x}_i \\
\dot{\ell} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & \frac{m_{t,\text{eff}}}{m_i} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_i \\
\ell \\
\theta
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{1}{m_{t,\text{eff}}} \\
0 \\
\frac{-1}{m_{t,\text{eff}}}
\end{bmatrix} F_i
\]

\[ \text{(4)} \]

**Newtonian model**

A container has six degrees of freedom (6 d.o.f.) for its movements. The container can show a translation in three directions and a rotation around three axes:

- trim,
- list,
- skew.

During the movement of the trolley the container will sway. For the reduction of this motion control algorithms are implemented and tested in the laboratory and in practice on real cranes. Under specific conditions the container can also show a skew rotation. Because of this accurate positioning of the container will be obstructed and therefore the skew has to be reduced.

For the description of these movements a 3D model (or a quasi-3D model) has been developed. This model describes the dynamics of the container suspended on four parallel cables. In this model it is allowed that the length of both cables is different, which may lead to skew. The centre of gravity of the container can also be different. This centre of gravity is an important parameter for the skew motion. Manipulating the length of the cables and/or its position has been considered as a possibility to influence the skew movement. Models, describing the dynamic behaviour of cranes including the skew motion are distinguished with respect to complexity by:

- models for which the length of all cables connected to the load are equal having either constant or variable length,
- models for which the length of all cables connected to the load are unequal again having either constant or variable length.
These models allow varying the connection of the suspension of the hoisting cables to the trolley. Depending on the complexity of the model we can distinguish between:
- models for which the length of all cables connected to the load, are equal and constant while the position of the suspension can be varied, and
- models for which the length of all cables connected to the load, are unequal and constant while the position of the suspension can be varied.

Figure 2 shows the configuration of the trolley and container system. In order to model the motion dynamics of the container body, two coordinate reference frames are assigned:
- a global reference frame on the base of the rail,
- a container attached reference frame originated in the geometric centre of the container.

It is clear from Figure 2 that the displacement vectors for the cables can be stated as

$$^oD_i = ^oA_i - ^oC_i$$ for $$i = 1 \cdots 4$$

The locations of the pulleys $$^oA_i$$ on the trolley can be derived from the trolley position on the rail $$x_T$$ and the geometric design of the crane. The locations of the pulleys $$^oC_i$$ on the container are derived from the motion of the container

$$^oC_i = \bar{x} + ^oR_C \cdot ^C C_i$$

where the pulley locations $$^C C_i$$ are in the local reference frame of the container. The forces in the cables are given by

$$^oF_i = \frac{^oD_i}{|^oD_i|} \left( K_S S_i + K_D \dot{S}_i \right)$$

where $$K_S$$ is the stiffness coefficient, $$K_D$$ the damping coefficient, and $$S_i$$ the stretch in cable $$i$$. With the unloaded cable length $$L_i$$, the stretch is computed as

$$S_i = \left| ^oD_i \right| - L_i$$

The total sum of the forces acting on the container and spreader is the sum of the individual cable forces, together with the wind and gravitational force. Since the 3-dimensional direction
of the forces is known, and also the locations where these forces act on the container (being the container pulleys \(O_C\)), also the torques on the container can be computed.

The resulting dynamic equations for the container mass can be represented by the diagram in Figure 3.

\[
\sum F_i + \bar{W} = M^{-1} \dot{\bar{x}} + \frac{1}{S} \ddot{\bar{x}} + \frac{1}{S} \dddot{\bar{x}} - \bar{G}
\]

Figure 3 Block diagram for the translational container motion dynamics.

A state space model description for the translational container motion \([2]\) can be expressed as

\[
\frac{d}{dt} \begin{bmatrix} \bar{x} \\ \bar{\dot{x}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{\dot{x}} \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} M^{-1} \sum F_i + \bar{W} \end{bmatrix} + G \end{bmatrix}
\]

where
- \(\sum F_i\) sum of cable forces,
- \(\bar{W}\) wind force in inertia reference frame,
- \(M\) mass matrix,
- \(M\bar{G}\) gravitational force,
- \(x\) position of centre of gravity c.g.

\[
^i (C_i \times \bar{x}) \times F_i + (C_i \times \bar{W})
\]

Figure 4 Block diagram for the rotational container motion dynamics.

The diagram in Figure 4 represents the model for the angular dynamics. The body rotation angles are the elements of \(\bar{\Omega}\). A state space model description for the angular dynamics \([2]\) is given by

\[
\frac{d}{dt} \begin{bmatrix} \bar{\Omega} \\ \bar{\dot{\Omega}} \end{bmatrix} = \begin{bmatrix} -I^{-1}SI & 0 \\ R_f & 0 \end{bmatrix} \begin{bmatrix} \bar{\Omega} \\ \bar{\dot{\Omega}} \end{bmatrix} + \begin{bmatrix} I^{-1}R_c \end{bmatrix} \bar{T}
\]

where
- \(\bar{T}\) torques acting on the container,
- \(\bar{\Omega}\) container gyroscopic angle,
- \(\bar{\Omega}\) Euler angle,
- \(\bar{\Omega} \times (I \times \bar{\Omega})\) coriolis and centrifugal torques,
Mechanical model

To get a better insight in the differences between the results of a computer simulation and an experimental setup of the crane, a mechanical model of a Jumbo Container Crane was developed and built. The laboratory model is a scale model of the Jumbo container crane (see Figure 5). The scaling factor is 33, so all dimensions are reduced with a factor of 33. The scale factors of the mass, speed, power, etc. have to be adapted.

Figure 5 Laboratory scale model of the crane.

The laboratory model will show qualitative, similar complicated effects like friction and elasticity in the cables and the flexibility of the mechanical construction.

In a quantitative sense, there are differences between the model crane and the Jumbo Container Crane. Besides the indicated effects are depending on temperature, wear and tear, maintenance, etc.

The laboratory model gives a better insight in the influence of these effects and makes it possible to compare the performance between the computer simulation with the applied mathematical model with the “practice” of a laboratory model. A number of effects can also be varied in the laboratory crane like for instance the flexibility of the mechanical construction.

3 TIME-OPTIMAL TRAJECTORIES

The main purpose of the control of a Jumbo Container Crane is to minimise the time for the transport of the container between the quay and the vessel or vice versa. The path to be followed by the container is the time-optimal trajectory. During the computation of this trajectory the boundaries like available motor power or maximal acceleration (maximum torque) and the maximum speed have to be taken into account. Also the obstacles on the quay or in the vessel will influence this trajectory.

The starting-point for the determination of the time-optimal trajectory is a container that is picked up from a stationary position (and sway angle zero) and is transported to a desired end-position, also with a sway angle zero. The real transport time of the container will be larger than the calculated optimal time, not only because the maximal acceleration and speed are not always available, but in particular because the optimal trajectory is not presenting a robust
solution. Going to the end-position takes additional time because some care have to be taken into account to reach the end-position within the desired accuracy (less than 3 cm and a sway angle zero) in spite of always appearing disturbances.

Several methods of optimisation are available to calculate the time-optimal trajectories:
- local optimisation,
- dynamic programming (Bellman),
- model-based predictive control,
- maximum principle of Pontryagin,
- genetic algorithm.

**Local Optimisation**
Local optimisation determines the optimal trajectory following an intuitive strategy. The control of the crane is using a sway angle of the container unequal to zero only during the period of acceleration (start of the movement) and the period of deceleration (end of the movement) (if there is no wind). There is a free choice for the curve of the acceleration in the initial phase and the deceleration of the final stage. For the local optimisation a profile for the acceleration is calculated for which the maximal attainable speed is reached in the shortest time under the condition that the sway angle will be zero at the end. The starting phase (the acceleration phase) is optimised explaining the term local optimisation. For the end-phase (the deceleration phase) there exist a similar opposite profile for the control if the friction and losses are neglected. Otherwise the deceleration phase have to optimised separately.

It is an advantage that the load will not swing forward and backward during the acceleration and deceleration phase to obtain a transport cycle that is more smooth and causes less wear of the gearbox in the crane. Figure 6 shows the result of a computer simulation of a trajectory for a distance of 45 m, a constant cable length $\ell=15$ m, a maximal speed $v=4.5$ m/s and a container mass $m=24000$ kg. The transport time is 16 s.

**Dynamic programming**
Dynamic programming is optimising the complete trajectory resulting in a faster transport time under equal conditions. The result of a computer simulation is presented in Figure 7 for the same conditions as for local optimisation. The transport time is practical the same: almost
16 s. Dynamic programming can calculate the time optimal trajectory also for avoiding obstacles. However, the necessary computation time is extremely large in particular for varying cable length (for constant cable length already 15 h). Besides the container swings backward and forward, which leads to extra wear of the gearbox of the crane.

Model-Based Predictive Control (MBPC)
Model-Based Predictive Control is applicable for both the generation of the time optimal trajectory and for following this trajectory. At the end of the time optimal trajectory the weight for the time-optimality can be reduced in favour of a more careful control strategy, for which a more robust and accurate positioning with a sway angle equal to zero could be reached with a low sensitivity to disturbances. Varying cable length during the start and the end of the container transport cycle including obstacles can be included during the following of the trajectory.

MBPC is based on making a prediction of the future optimal input for the control of the crane. The reach of this prediction is indicated by the value of the prediction horizon. To limit the number of calculations the controller changes its strategy only during a specific fixed control horizon. After that the output of the controller stays constant during the remaining part of the prediction horizon.

Several MBPC systems are examined among others with constant and with variable sampling frequency and with constant and variable prediction horizon. In Figure 8 the results of a simulation is shown as a time-optimal trajectory for a transport of a container given some obstacles on the quay.

A disadvantage of MBPC for the estimation of the optimal trajectory is the large computation time.

The control of a MBPC is based on a model of the crane. It is not necessary that this model is accurate to obtain an accurate following of a specific trajectory because for each sample the control signal is renewed. The positioning of the container at the end however must be very accurate, independent of the applied model. For the end positioning special attention have to be payed to the accuracy of the controller.
The theory shows that a bang-bang controller is able to time-optimal control of a process. The results of this method are applied to the process of transport of a container. Figure 9 shows the result for a constant cable length $\ell$ [4]. This figure shows that a container of $m_C=47000$ kg can be transported in 14 s over a distance of $x_C=45$ m. The weight of the trolley is 33000 kg. The maximal torque of the trolley drive is 4200 Nm, the maximal torque of the hoisting drive is 11490 Nm while the maximum speed of the trolley is 4.5 m/s.

This method is now very useful as a benchmark for judging the results of more practical implementations of a controller.

The computation time is about equal to the transport time of the container.

Figure 9 Time-optimal trajectory for constant cable length.
CONTROL SYSTEM

The control system is composed of two controllers:
- a sway controller,
- a skew controller.

The purpose of the sway controller is to generate and follow real-time the time-optimal trajectory and to position the container in the end-position with a sway angle zero and within the required accuracy. The controller takes into account the constraints of the crane design. The skew controller will eliminate the skew angle of the container as fast as possible.

Both controllers are designed as a state-feedback controller with variable feedback gains and an on-line gain optimizer. Both controllers need information from the sensors assembled with the crane.

4.1 SWAY CONTROL

Figure 10 presents a block diagram of the container crane with the control system. The control system consists of two main parts: a state feedback loop with time variable feedback gains ($k_1$ through $k_4$) and a gain optimizer.

The gains are optimised by computing the natural frequency of the closed loop system as a function of time that minimises a performance index, subject to constraints. In order to compute the value of the performance index and the constraints, the future behaviour of the container crane is predicted using a model of it.

**Time Variable Feedback Gains**

To improve the performance of the control system, the feedback gains have to be time variable for three reasons. The first reason is that the cable length $\ell$ and the mass of load $m$ change during loading and unloading of a container vessel. In order to achieve fast and accurate performance of the container crane the feedback gains have to be adapted for different $m$ and updated when $\ell$ changes.

The second reason is that the force $F_t$ of the trolley drive system is limited. For time optimal performance the container crane should always operate close to or at the maximum force $F_{t,\text{max}}$. But when the four feedback gains are small relative to the error signals ($e_1$ through $e_4$) the controller output ($F_{t,\text{d}}$, see Figure 10) is much smaller than $F_{t,\text{max}}$, so that the response is not time optimal. On the other hand, when $k_1$ through $k_4$ are big relative to $e_1$ through $e_4$ the
limitation of \( F_t \) can cause large overshoot or even instability. Therefore, in order to achieve time optimal and accurate performance, while avoiding overshoot, it is necessary to use time variable feedback gains.

The third reason is that the trolley speed and the acceleration should not exceed a specified maximum value (\( \dot{x}_t, \text{max} \) and \( \ddot{x}_t, \text{max} \), respectively). By varying the values of the feedback gains it is possible to influence the trolley speed and the acceleration to avoid violation of the constraints.

The four feedback gains are tuned in order to obtain a deadbeat response [2]. The natural frequency of the resulting closed loop system is optimised. In order to compute the value of the performance index and the constraints, \( x_c, \dot{x}_t, \text{max}, \ddot{x}_t, \text{max} \), and \( F_t \) are predicted, given the current state and a model of the crane. A prediction model can be based both on the linear and the non-linear model, but in order to speed up the optimisation the linear model (equation 4) is used.

The performance index and the constraints are non-linear functions of the parameters that have to be optimised (\( c_0, c_1, \text{and } t_{\text{join}} \)). Therefore, the optimisation algorithm should be able to solve non-linear optimisation problems with non-linear constraints.

4.2 **SKEW CONTROL**

As mentioned before, by controlling the position of the suspension of the cables the skew can substantially be reduced.

In the construction of the container crane the cables are connected to two wagons placed on top of the trolley as schematically is shown in Figure 11. These wagons (wagon 1 and wagon 2) can be moved with respect to the trolley forward and backward in opposite direction. In this way the skew can be effectively eliminated.

For controlling the skew motion of the container, a similar approach has been applied as for the sway control, that was described above. However, in contrary with the gain optimization...
algorithm for the sway controller, a multi-dimensional lookup table has been generated for the four feedback gains. An additional unknown parameter that characterizes the skewing motion of the container is its inertia. The inertia is estimated from the size of the container and an empirical value, that has been quantified for a homogeneously loaded container for each size. At each time step the values for the gains are looked up and passed to the skew controller based on the cable length and inertia.

Figure 12 presents a block diagram of the container crane with the control system. The control system consists of two main parts: a state feedback loop with time variable feedback gains ($K_1$ through $K_4$). The skew and sway controller are based on similar design principles.

5 RESULTS
Simulation results swing control
For all results sofar the container crane is simulated with the non-linear model (section 2) and the parameters of the Jumbo Container Crane (JCC-2000, see table). This container crane is being developed within the FAMAS program.

Table: Simulation parameters of the JCC-2000

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum trolley drive system force</td>
<td>80 kN</td>
</tr>
<tr>
<td>Maximum overshoot container position</td>
<td>0.05 m</td>
</tr>
<tr>
<td>Maximum trolley speed</td>
<td>4.33 m/s</td>
</tr>
<tr>
<td>Maximum trolley acceleration</td>
<td>1.5 m/s²</td>
</tr>
<tr>
<td>Mass of trolley</td>
<td>35000 kg</td>
</tr>
<tr>
<td>Effective mass of trolley</td>
<td>53571 kg</td>
</tr>
</tbody>
</table>
The control system can be taken into operation without the need for an intricate tuning procedure. Only $m_{t,\text{eff}}$ and the constraints ($x_{t,\text{max}}$, $\dot{x}_{t,\text{max}}$, $x_{c,\text{os,max}}$ and $F_{t,\text{max}}$) have to be specified. All other parameters needed for control can be measured on-line.

The control system can be applied in several ways. It can be used for fast and accurate positioning of the container near the end position or it can be used to cover the complete trajectory. Figure 13 presents results for a complete trajectory, with initial state $[45 \ 0 \ 0 \ 0]$, mass of load 40.000 kg, cable length 15 m.

**Figure 13**: Simulation results for the JCC-2000.
Initial state: $\mathbf{x} = [x \ \dot{x} \ \theta \ \dot{\theta}] = [45 \ 0 \ 0 \ 0]$, mass of load 40.000 kg, cable length 15 m.

The control system can be taken into operation without the need for an intricate tuning procedure. Only $m_{t,\text{eff}}$ and the constraints ($x_{t,\text{max}}$, $\dot{x}_{t,\text{max}}$, $x_{c,\text{os,max}}$ and $F_{t,\text{max}}$) have to be specified. All other parameters needed for control can be measured on-line.

The control system can be applied in several ways. It can be used for fast and accurate positioning of the container near the end position or it can be used to cover the complete trajectory. Figure 13 presents results for a complete trajectory, with initial state $[45 \ 0 \ 0 \ 0]$, mass of load 40.000 kg, and $l=15$ m. The trolley speed and acceleration do not exceed the maximum value, the overshoot equals 2.3 cm and the settling time 15.5 s.

**Simulation results skew control**
A controller based on the model shown in Figure 11 is designed and tested. The basis of this model-based controller is equal to the trajectory controller as developed for the time-optimal transport of the container.
In Figure 14 shows the results of a simulation of the skew controller for different cable length between 10 m and 50 m. From the simulation it follows that an initial skew angle of 5° and a cable length of $\ell=50$ m is eliminated within 5 s.

![Figure 14 Results of simulation of the skew controller for different cable length.](image)

Figure 15 shows the results of a simulation in which there is a combined skew and sway movement. This figure shows how the skew angle is also reduced to zero in a short time even if there is a significant sway. The maximum swing angle is here 10° while the skew angle of 5° is eliminated within 11 s for a cable length of $\ell=35$ m.

![Figure 15 Results of simulation of a skew controller for a combined skew and sway motion.](image)

Figure 15 shows the results of a simulation in which there is a combined skew and swing movement. This figure shows how the skew angle is also reduced to zero in a short time even if there is a significant sway. The maximum swing angle is here 10° while the skew angle of 5° is eliminated within 11 s for a cable length of $\ell=35$ m.
**Results of scaled laboratory model**

Figure 16 shows the result of the applied sway controller to a container with a mass of $m_C = \frac{35000}{33^2} = 0.974$ kg and a constant cable length $\ell = \frac{53}{33} = 1.6$ m. The programmed trajectory (top figure) is followed closely and the container arrives in the end position in 15 s. For the end position the swing angle (bottom figure) is zero. Similar result are obtained for $0.3 < \ell < 1.9$ m.

![Figure 16](image1.png)

**Figure 16** Following a prescribed trajectory:
- Top: calculated and measured trolley position,
- Bottom: swing angle.

![Figure 17](image2.png)

**Figure 17** Trajectory for a state-feedback controller.

Figure 17 shows the trajectory for a state feedback controller.

The performance of the controller is also conforme to the specifications for JCC-2000 scaled to the mechanical model as follows:
1. Transfer time for 1502 mm distance $\leq 4.4$ sec
2. Accuracy at the end position $\leq 1.5$ mm
3. Swing at the end position $\leq 0.00075$ rad ($0.043^\circ$)
   The first is not satisfied, while the second and third specifications are satisfied.

6 CONCLUSIONS
A control system for a container gantry crane has been presented. The controller is based on a state feedback loop with time variable gains, which are optimised using a model based optimisation strategy.

By applying this controller the crane position a container in a fast way without swing an skew from any random initial conditions and for any arbitrary mass or load and cable length. The positioning of the container is accurate and within the required 5 cm.

For the transport of the container between the quai and the vessel or vice versa different methods for determining the time optimal trajectory are investigated. Until now Pontryagin’s maximum principle seems to be the most useful for real-time applications.

The control system is tested with different linear and non-linear models in a computer environment as well as on a scaled mechanical laboratory model of the crane. On this laboratory model it has been checked that the control system can easily be taken into operation.

The transport of the container has been studied also for different constraints and disturbances. The gain in the time for transporting a container resulting from doubling of the maximal torque and increasing the speed with 50% seems to be insufficient to justify the high additional investment and operational costs.

The controller also compensates for disturbances like wind forces and keeps the positioning of the container within the requirements.

REFERENCES