Internal Impedance Characterization for Black Box Signal Source

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Abstract

Different system identification techniques for using experimental data to characterize the waveform and internal impedance of a voltage source are examined. In the automotive industry, an Interference Signal Generator (ISG) is used to synthesize Electro Static Discharge (ESD) spike pulses according to SAE specifications (SAE Test Pulses 1,2,3a & 3b) [1]. The project[1] involves high speed measurement and data manipulation of these transient spikes and modeling, simulation and validation of an equivalent model for the signal source. Using the result, an electric circuit that accurately behaves like the internal impedance of the signal generator was implemented in a SABER[2] model. The experimentally validated model is used for simulation of automotive protection circuits.

Keywords: Identification and Model Reduction, Automotive, Industry day.

1 Introduction

In recent years, the Chrysler Corporation has pursued SABER as a standard for modeling and simulation on their vehicle circuits components and subsystems. In many cases, expensive experiments can be substituted by computer simulations and experimental results and possible failures can be predicted before hardware is built.

This paper describes a SABER model for Electro Static Discharges (ESD) and SAE test pulses. Since many research experiments on breakdown tests will be performed using a spike generator, there is a need to formulate an accurate model for this equipment available in the SABER library for the purpose of simulation.

In this way, a model for the generator can be pulled from the library and simulation can be performed with a push on a button. This paper describes the process we took, to synthesize an experimentally validated signal source model.

[1] The project was conducted in collaboration with and supported by Chrysler Corporation.
[2] SABER is a simulation and analysis tool for analog, digital and mechanical systems, [3].

2 Problem Description

The Interference Signal Generator (ISG), provides the capability to select the magnitude and energy of the pulse to be generated, so that it confirms to the SAE specifications. Figure 1 shows a general Thevenin equivalent model for the power source. The goal is conduct a set of experiments to determine the nominal spike voltage V_s and the source impedance Z_s. The problem is to identify V_s and Z_s with only the measurements at the terminals of the ISG; these being the voltage V_o across and the current I_o through the load impedance. The area in the dashed box in Figure 1 is not accessable for any measurements. We will connect load impedance, Z_l, with different characteristics, as shown, onto the terminals of the generator to get sufficient information in the measurements that we need to solve the identification problem.

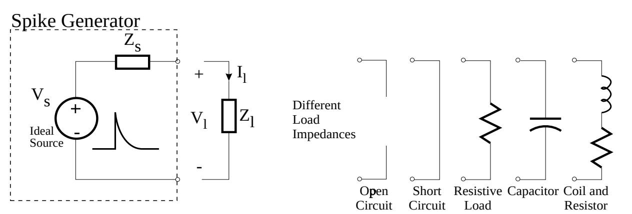

Figure 1: *Model of a Spike Voltage Source.*

3 Lab Setup

The right side of Figure 1 shows the different load impedances that were used in the experiment. Figure 2 shows the experimental setup for capturing the waveforms onto the computer.

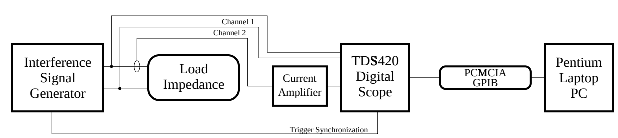

Figure 2: *Configuration of the experimental setup.*

The current and the voltage were measured and aquired in a Texas Instruments digital scope TDS420 with GPIB capabilities. A sampling time of $T_s = 0.1\ \mu s$ (or sampling frequency of $f_s = 10\ MHz$) was used for the aquisition. A laptop computer

with a GPIB-PCMCIA plug-in card was connected to the scope to download the data-file into the computer [6].
Typical waveforms of the spike voltage V_{open} and current I_{short} are drawn in Figure 3. Of course the current of an open-circuit experiment, as well as the voltage of a short-circuit experiment show no information. Many more sets of experiments were conducted with different load impedance settings.

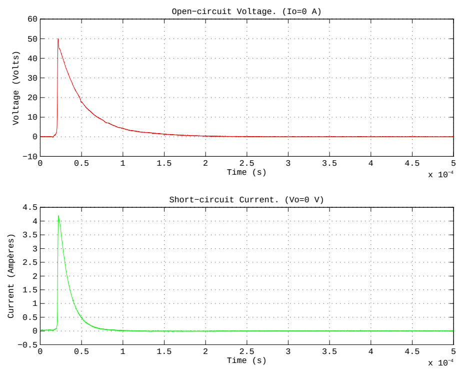

Figure 3: *Typical transients of the open-circuit voltage and the short-circuit current.*

4 Identification Methods

Once the experimental data have been aquired and converted into the appropriate format, MATLAB was used to conduct the signal processing and identification. A graphical user interface was developed in MATLAB for the identification process. Several tools are available in MATLAB to calculate in, and transfer across different representation domains. Figure 4 shows the domains and the MATLAB commands for the conversions.

Three methods are described below. Information from these methods will be used to arrive at a transfer function that characterizes the internal impedance of the ISG.

4.1 FFT Non-parametric ID

Consider the source and load diagram shown in Figure 1. Expressed in the Laplace domain, the source V_s and load voltage V_l, the internal impedance $Z_s(s)$, and the load impedance and $Z_l(s)$ and the output current $I_l(s)$.

$$V_s(s) = I_l(s)Z_s(s) + V_l(s) \tag{1}$$

$$V_l(s) = I_l(s)Z_l(s) \tag{2}$$

If no load is connected to the terminals, no current flows through the network $(I_l(s) = 0)$. In this case, the measured voltage across the terminals will be $V_{open} = V_s$. In the case where the load impedance is zero the output voltage $V_l(s) = 0$, and we can measure the short-circuit current as $I_{short} = \frac{V_s}{Z_s}$. The internal impedance can then be determined as

$$Z_s(s) = \frac{V_{open}(s)}{I_{short}(s)} \tag{3}$$

Next denote the sampled open-circuit voltage as $V_o(k) = V_{open}(kT)$, where T is the sampling interval and $k = 1, 2, \ldots, N$, N being the total number of samples. Similarly, denote $I_s(k) = I_{short}(kT)$. The Discrete Fourier Transformation (DFT) maps the sampled voltage and current into discrete frequency representation using equations (4) and (5).

$$V_N(\theta_n) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} V_o(k)e^{-i\theta_n k} \tag{4}$$

$$I_N(\theta_n) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} I_s(k)e^{-i\theta_n k} \tag{5}$$

where N holds the amount of samples in the measurement data and

$$\theta_n = \frac{2\pi n}{N} \qquad n = 1, 2, \ldots, N-1 \tag{6}$$

are the discrete frequency angles. Note that the DFT yields the efficient Fast Fourier Transform (FFT) when N is of radix 2. Following (3), the discrete frequency response function for the internal impedance can be written as

$$\hat{Z}_N(\theta_l) = \frac{V_N(\theta_l)}{I_N(\theta_l)} \tag{7}$$

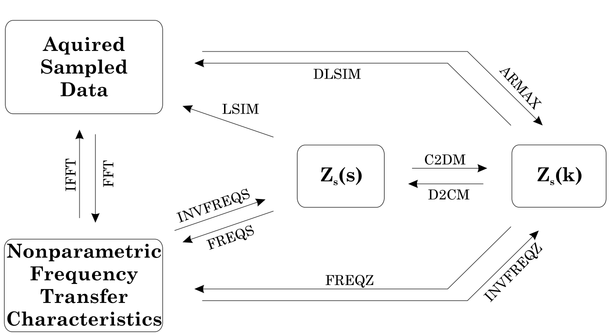

Figure 4: *Several tools are provided in MATLAB, to transfer across different domains.*

Figure 6 shows the identified model for the internal impedance as a curve in frequency domain. It is calculated in MATLAB with the following sequence of commands.

```
time = (1:5000) * 0.1e-6;
fftvo = fft(vo);
fftis = fft(is);
ff = time(2:2500) / max(time) * 2*pi;
fftzi = fftvo ./ fftis;

loglog(ff,abs(fftzi(2:2500)));
loglog(ff,angle(fftzi(2:2500)));
```

In this example `vo` and `is` hold the measurement data for the open-circuit voltage and the short-circuit current respectively. The functions `fft`, `max`, `abs`, and `angle` are MATLAB supported. Note that we only need to display the data up to the Nyquist frequency.

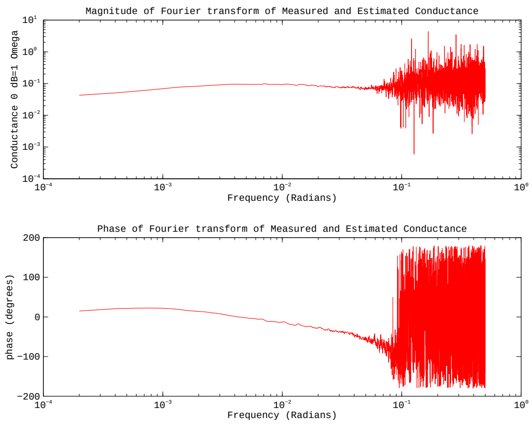

Figure 5: *Non-parametric frequency domain characterization of the internal conductance* $\frac{1}{Z_s(s)}$.

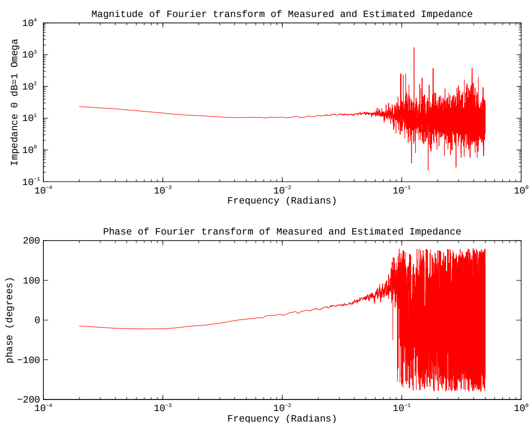

Figure 6: *Non-parametric frequency domain characterization of the internal impedance* $Z_s(s)$.

4.2 FFT Parametric ID

Identification based on non-parametric frequency response characteristics is called the *Emperical Transfer Function Estimate (ETFE)* [4]. From the non-parametric model, a polynomial model can be identified. This model will be the transfer function representation for Z_s. The identification is performed in MATLAB, using the command `invfreqz` from the Signal Processing Toolbox.

The command uses a fast algorithm to estimate a discrete transfer function that corresponds to the FFT of Z_s in Equation (7). With na and nb given , it converts the magnitude and phase data shown in Figure 6 into a transfer function of the form

$$\begin{aligned} Z_s(z) &= \frac{B(z)}{A(z)} \\ &= \frac{b_1 + b_2 z^{-1} + \ldots + b_{nb+1} z^{-nb}}{a_1 + a_2 z^{-1} + \ldots + a_{na+1} z^{-na}}. \end{aligned} \quad (8)$$

The algorithm, explained in more detail in [7], uses an equation error method to identify the best model for the data.

If the complex frequency response is given in `h` at frequency

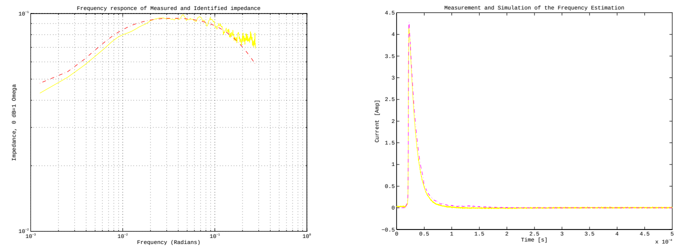

Figure 7: *The curve fitting in frequency domain and a simulation of the model with the open-circuit data.*

points specified in `w`, then from the MATLAB command line the algorithm is invoked as

```
[b,a] = invfreqz(h,w,nb,na)
```

and will return the $A(z)$ and $B(z)$ polynomial coefficients and also plot the frequency response of input data and simulation of th identified model in one graph. By default `invfreqz` uses an equation error method to identify the best model for the data. This finds $B(z)$ and $A(z)$ in

$$\min_{B(z),A(z)} \sum_{k=1}^{n} \theta_n(k) \left| \hat{Z}_N(k) - \frac{B(\theta_n)}{A(\theta_n)} \right|^2 \quad (9)$$

by creating a system of linear equations and solving them with the least squares method. Here $A(\theta_n)$ and $B(\theta_n)$ are the Fourier Transforms of the polynomials $A(z)$ and $B(z)$ in Equation (8) at the frequency θ_n, and n is the number of frequency points. ω_t is a vector of weights that weight the error approximately relative to the error at different frequencies.

Non-convergent result in identifying the impe-dance model. Applying this method to the current problem, the vector $\hat{Z}_N(\theta_n)$ holds the non-parametric representation of the discrete frequency response for the internal impedance as given in Equation (7) and shown in Figure 6. $A(\theta_n)$ and $B(\theta_n)$ represent a frequency response of a parametrical model for the internal impedance $Z_s(z)$ as given in (8).

No convergence could be obtained when identifying a model for the internal impedance with this data. This can be understood by looking at Figure 6 and recognizing the trend at high frequencies. This can be interpreted as $Z_s(z)$ being a non-proper function, whereby the polynomial $B(z)$ is of higher order than $A(z)$. A proper filter should attenuate at high frequency.

Convergent result in identifying the conductance model. The fraction in Equation (7) was flipped, and this results in a frequency response for the conductance $\hat{G}_N(\theta_n)$, as shown in Figure 5. Now, identification will converge towards a proper transfer function that represents the conductance $\hat{G}_s(z)$.

In Figure 7 a comparison between the estimated parametric model and the empirical frequency response data is displayed on the frequency axis. It is ensured that no frequencies above the Nyquist frequency are used for identification. With a $0.1\ \mu s$ sampling interval, the valid frequency bandwidth is 5 MHz. Only a much lower frequency band is selected for identification since there is a lot of noise in the higher bands. In the right graph

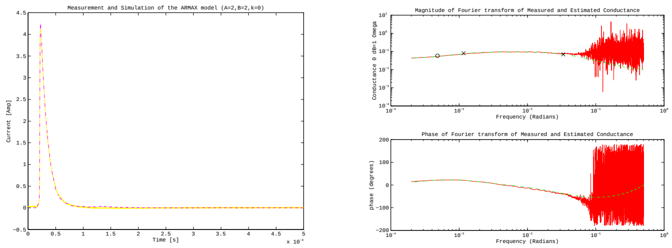

Figure 8: *The short-circuit current and the output of the ARMAX-transfer function, together with the frequency information in both signals.*

of Figure 7, both the simulation of the identified model and the closed-circuit data, that was measured from the experiments in the lab, are displayed. It can be seen that the simulation result matches well with the measurement data, and hence confirm the valadity of the conductance model $\frac{1}{Z_s(z)} = \frac{A(z)}{B(z)} = \hat{G}(z)$.

4.3 Parametric ID (ARMAX)

The ETFE is a rough estimate since the data series of N inputs and N outputs is reduced to a series of $N/2$ complex numbers that describe the transfer function. And for identification only less than 300 frequency points were used to estimate the transfer function.

An auto regressive moving average filter will be estimated in time domain. The general shape of the so called ARMAX models is as follows,

$$A(z)y(k) = B(z)u(k) + C(z)e(k) \tag{10}$$

where $A(z)$, $B(z)$ and $C(z)$ stand for polynomials in the discrete time domain. For example

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_n z^{-n} \tag{11}$$

If we collect the parameters of the polynomials as $\theta = (a_1, \ldots, a_{n_a}, b_1, \ldots, b_{n_b}, c_1, \ldots, c_{n_c})$ and define

$$G(z,\theta) = \frac{B(z)}{A(z)} \quad H(z,\theta) = \frac{C(z)}{A(z)}$$

then it can be shown that the step-ahead predeiction

$$\begin{aligned}\hat{y}(k|k-1) =& H^{-1}(z,\theta)G(z,\theta)u(k) + \\ & \left(1 - H^{-1}(z,\theta)\right) y(k)\end{aligned} \tag{12}$$

and the ARMAX prediction model follows as

$$\begin{aligned}\hat{y}(k|\theta) =& \frac{B(z)}{C(z)}u(k) + \\ & \left(1 - \frac{A(z)}{C(z)}\right) y(k)\end{aligned} \tag{13}$$

Defining the prediction error as $\epsilon(k|\theta) = y(k) - \hat{y}(k|\theta)$, then Equation (12) can be rephrased as

$$\begin{aligned}\hat{y}(k|\theta) =& B(z)u(k) + (1 - A(z)y(k) + \\ & (C(z) - 1)\epsilon(k|\theta)\end{aligned} \tag{14}$$

From here a regression vector will be selected as

$$\begin{aligned}\phi(k|\theta) = (& -y(k-1) \ldots -y(k-n_a) \\ & u(k-1) \ldots u(k-n_b) \\ & \epsilon(k-1|\theta) \ldots \epsilon(k-n_c|\theta))^T\end{aligned} \tag{15}$$

The ARMAX prediction model can now be rewritten in the following regresion form

$$\hat{y}(k|\theta) = \theta^T \phi(k|\theta) = \phi^T(k|\theta)\theta \tag{16}$$

Since $\hat{y}(k|\theta)$ is not a linear function of θ, this ARMAX prediction model is called a *pseudo linear regression model.*

Using this method, a similar transfer function $\frac{1}{Z_s(z)}$ will be identified. In the present context, the input and the output for the ARMAX will be set to $u(k) = V_o(k)$ and $y(k) = I_s(k)$ from the sampled experimental results. For more details about ARMAX parametric identification methods, [4] is referred to.

The System Identification Toolbox in MATLAB provides a complete automated script that implements the ARMAX prediction error method to identify the parametric model from measured transient data. A typical session from the command line in MATLAB looks like

```
z  = [y u];
nn = [na nb nc nk];
th = armax(z,nn);
[B,A] = th2tf(th);
yh = idsim(u,th);
plot([y yh]);
```

Figure 8 shows the result of one experiment with this algorithm. It can be seen that the time response matches the original output signal much more accurate. Parametric models for all input-output sets are derived and listed in Table 1. The bottom line in the table shows the average of the coefficients. This is allowed since the polynomials are linear. The left column lists the amperage per each experiment.

Exp.	a_2	a_3	b_1	b_2	b_3
9 A	$2.2\ 10^6$	$1.5\ 10^{11}$	-0.02	$4.1\ 10^5$	$1.2\ 10^{10}$
18 A	$3.0\ 10^6$	$2.3\ 10^{11}$	-0.03	$5.8\ 10^5$	$1.9\ 10^{10}$
27 A	$3.6\ 10^6$	$2.7\ 10^{11}$	-0.03	$6.9\ 10^5$	$2.3\ 10^{10}$
36 A	$3.7\ 10^6$	$2.6\ 10^{11}$	-0.04	$7.0\ 10^5$	$2.3\ 10^{10}$
27 A	$4.0\ 10^6$	$2.8\ 10^{11}$	-0.04	$7.4\ 10^5$	$2.3\ 10^{10}$
av	$3.3\ 10^6$	$2.4\ 10^{11}$	-0.03	$6.1\ 10^5$	$1.9\ 10^{10}$

Table 1: *Identified parameters for five different experiments.*

4.4 Model Reduction

From the preceding section, it turns out that the ARMAX algorithm yields a more accurate estimated model, which will be used from here as the identified transfer function. Using the MATLAB command `d2cm` (See Figure 4), the transfer function of the identified conductance can be expressed in the s-domain [2]. The internal conductance becomes as

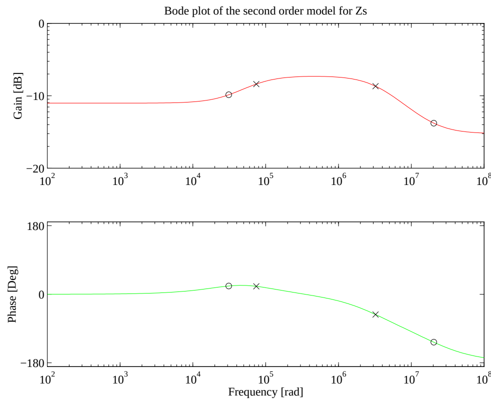

Figure 9: *The Gain and Phase of the second order model for the internal conductance* $\hat{G}(z)$.

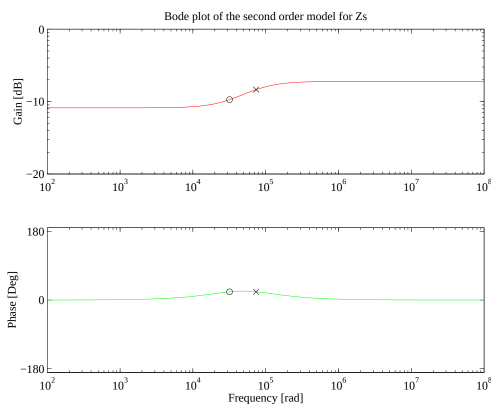

Figure 10: *The Gain and Phase of the first order reduced model for* $\hat{G}(z)$.

$$\frac{1}{Z_s(s)} = \frac{-0.03s^2 + 6.1\ 10^5 s + 1.9\ 10^{10}}{s^2 + 3.3\ 10^6 s + 2.4\ 10^{11}} \tag{17}$$

$$= -0.03\frac{(s - 2.0\ 10^7)(s + 3.2\ 10^4)}{(s + 3.2\ 10^6)(s + 7.4\ 10^4)}. \tag{18}$$

Figure 9 shows the Bode plot for the transfer function, together with the zeros frequencies, represented by circles and the poles represented by crosses.

It can be seen from the figure that there is a pole and a zero that are very close to the sampling frequency and therefore does not contribute to the characteristics in the valid frequency range. These roots could be neglected compared with the other roots. Model reduction of Equation (18) thus yields

$$\frac{1}{Z_s(s)} = 0.19\frac{(s + 3.19\ 10^4)}{(s + 7.40\ 10^4)}. \tag{19}$$

Figure 10 shows the Bode plot with gain and phase of this reduced model. Again the pole and zero are marked in the plots. Notice that the shape of the curves on the left of 1 MHz equals the shape of the curves in the Bode diagram of Figure 9.

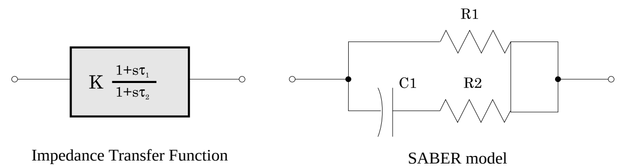

Figure 11: *A representation of a first order transfer function, representing the reduced order model that was identified from the measurement data.*

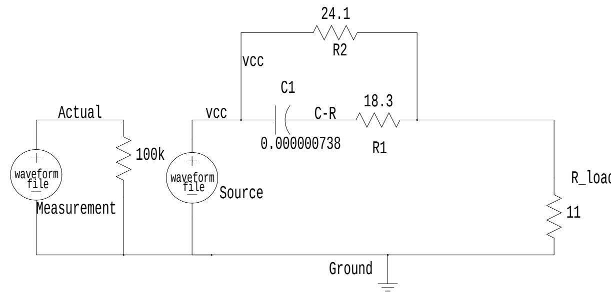

Figure 12: *The SABER schematic with the R-C-circuit that implements the identified* Z_s.

5 Resulting Circuit

The estimated model for the internal impedance Z_s, given in Equation (19) can be represented by many passive electrical circuits. Here we choose a representation in a R-C-circuit. It should be stated that this representation is not unique and may not represent the actual internal circuit of the Spike Generator. A general first order filter in Laplace domain is of the form

$$Z_s(s) = K\frac{1 + s\tau_1}{1 + s\tau_2}, \tag{20}$$

as shown in Figure 11. The impedance of the circuit in the right of the figure equals

$$Z_s(s) = R_1\frac{(1 + sC_1R_2)}{(1 + sC_1(R_1 + R_2))} \tag{21}$$

To match the first order model with the system, the three parameters of the filter function in Equation (20) should be estimated as

$$K = R_1 = 12.23 \tag{22a}$$

$$\tau_1 = C_1R_2 = \frac{1}{73900} \tag{22b}$$

$$\tau_2 = C_1(R_1 + R_2) = \frac{1}{31972}. \tag{22c}$$

The values for the elements in the R-C-circuit can be estimated as $R_1 = 12.23\ \Omega$, $R_2 = 18.3\ \Omega$, and $C_1 = 0.738\ \mu F$.

The identified internal impedance carries information on how a nominal signal from the ideal source is perturbed by adding load on the terminals.

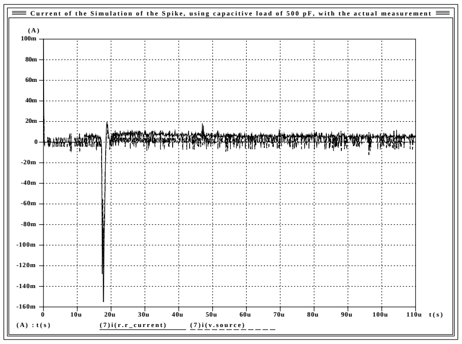

Figure 13: *Validation with a capacitive load of* $500pF$.

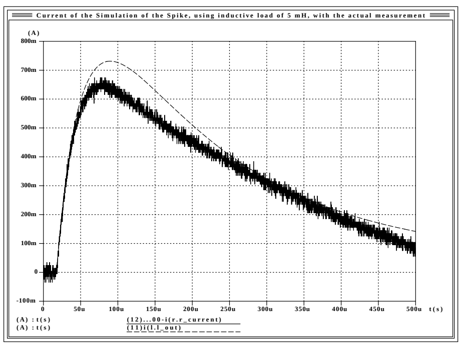

Figure 14: *Validation with an inductive load of* $5mH$.

6 Validation

To validate the model, different load impedances were tested. The load currents and voltages for each load impedance were recorded. Then a SABER simulation was conducted with an ideal spike wave-form, characterized by the settings on the front of the generator equipment. Similar circuits as in Figure 12 were setup with the appropriate load impedances for `R_load`. Two experiments are shown here. Figure 13 shows the simulation and the experimental transient of a capacitance of 500 pF, connected to the ISG. The traces of the two trasient responses were not distinguishable. Figure 14 shows the simulation and the experimental transient of a capacitance of 5 mH, connected to the ISG. The latter experiment is slightly off. This is due to the internal resistance of the experimental inductance array box that was not modeled in the simulation. The resistance turned out to be relatively large. This problem illustrates the difficulty of obtaining experimentally validated simulation.

7 Conclusions

Using informations from both time and frequency domains adds to the understanding of the measurement data and the estimated model. The ETFE showed that a conductance had to be identified and that sampling information up to 500 kHz was valid for identification. The time domain identification method generated a 2 pole - 2 zero model in the Z-domain as the least order system to simulate the internal impedance Z_s. Model reduction technique can be applied to lower the order of the model. Based on the results from these methods, a physical component circuit can be formulated to match measurement data well. In the simulation, the designer is now able to "grab the validated ISG model from the shelf" and conduct simulation experiments, to obtain simulated data that accurately compares to the same experiment in the lab.

References

[1] Chrysler. Chrysler corporation engineering standard. Technical report, PF-9326.

[2] G.F. Franklin and J.D. Powell. *Digital Control of Dynamic Systems*. Addison-Wesley, 1980.

[3] Analogy Inc. *SABER Manuals and Library References*. P.O. Box 1669, Beaverton, OR 97075-1669.

[4] L. Ljung. *System Identification: Theory for the user*. Prentice-Hall, 1987.

[5] L. Ljung, J.N. Little, and L.Shure. *System Identification Toolbox, Signal Processing Toolbox, MATLAB Reference Guide*. The MathWorks Inc.

[6] National Instruments, 6504 Bridge Point Parkway, Austin, TX 78730-5039. *Getting started with your PCMCIA-GPIB and the NI-488.2 software for Windows*, May 1994.

[7] J. O. Smith. *Techniques for Digital Filter Design and System Identification with Application to the Violin.* PhD thesis, Stanford University, June 1983.